Lesson 10: Unknown Angle Proofs—Proofs with Constructions

Student Outcome

- Students write unknown angle proofs involving auxiliary lines.

Lesson Notes

On the second day of unknown angle proofs, students incorporate the use of constructions, specifically auxiliary lines, to help them solve problems. In this lesson, students refer to the same list of facts they have been working with in the last few lessons. What sets this lesson apart is that necessary information in the diagram may not be apparent without some modification. One of the most common uses for an auxiliary line is in diagrams where multiple sets of parallel lines exist. Encourage students to mark up diagrams until the necessary relationships for the proof become more obvious.

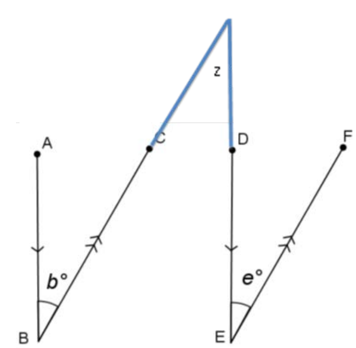
Classwork

Opening Exercise (7 minutes)

Review the Problem Set from Lesson 9. Then, the whole class works through an example of a proof requiring auxiliary lines.

Opening Exercise

In the figure to the right, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$. Prove that $b = e$. (Hint: Extend $\overline{BC}$ and $\overline{ED}$.)



Proof:

$b = z$ *If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*

$z = e$ *If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*

$b = e$ *Transitive property*

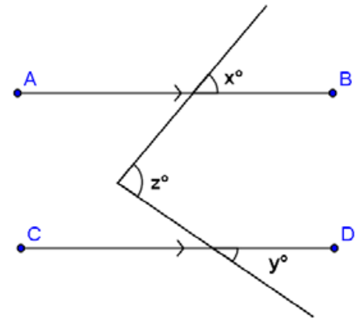
In the previous lesson, you used deductive reasoning with labeled diagrams to prove specific conjectures. What is different about the proof above?

Drawing or extending segments, lines, or rays (referred to as auxiliary lines) is frequently useful in demonstrating steps in the deductive reasoning process. Once $\overline{BC}$ and $\overline{ED}$ were extended, it was relatively simple to prove the two angles congruent based on our knowledge of alternate interior angles. Sometimes there are several possible extensions or additional lines that would work equally well.

For example, in this diagram, there are at least two possibilities for auxiliary lines. Can you spot them both?

Given: $\overline{AB} \parallel \overline{CD}$.

Prove: $z = x + y$.



Discussion (9 minutes)

Students explore different ways to add auxiliary lines (construction) to the same diagram.

Discussion

Here is one possibility:

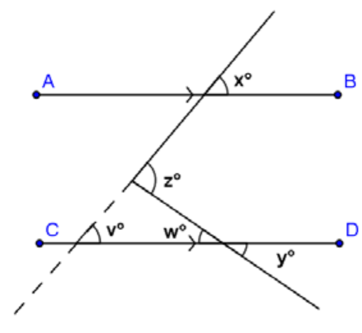
Given: $\overline{AB} \parallel \overline{CD}$.

Prove: $z = x + y$.

Extend the transversal as shown by the dotted line in the diagram. Label angle measures v and w, as shown.

What do you know about v and x?

About w and y? How does this help you?



Write a proof using the auxiliary segment drawn in the diagram to the right.

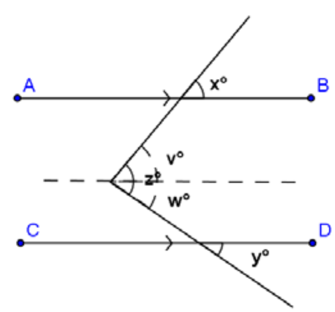
MP.7

$z = v + w$	*Exterior angle of a triangle equals the sum of the two interior opposite angles (remote) interior*
$x = v$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure*
$y = w$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure*
$z = v + w$	*Angle addition postulate*
$z = x + y$	*Substitution property of equality*

Another possibility appears here:

Given: $\overline{AB} \parallel \overline{CD}$.
Prove: $z = x + y$.

Draw a segment parallel to $\overline{AB}$ through the vertex of the angle measuring z degrees. This divides it into angles two parts as shown.



What do you know about v and x?

They are equal since they are corresponding angles of parallel lines crossed by a transversal.

About w and y? How does this help you?

They are also equal in measure since they are corresponding angles of parallel lines crossed by a transversal.

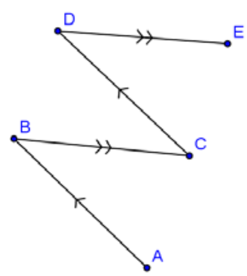
Write a proof using the auxiliary segment drawn in this diagram. Notice how this proof differs from the one above.

$x = v$	*If parallel lines are cut by a transversal, the corresponding angles are equal.*
$y = w$	*If parallel lines are cut by a transversal, the corresponding angles are equal.*
$z = v + w$	*Angle addition*
$z = x + y$	*Substitution*

Examples (25 minutes)

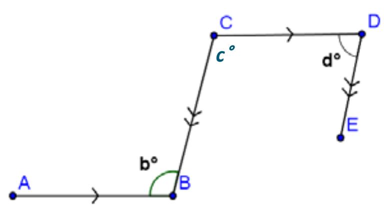
Examples

1. In the figure at the right, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$.
   Prove that $m\angle ABC = m\angle CDE$.
   (Is an auxiliary segment necessary?)



$m\angle ABC = m\angle BCD$		*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$m\angle BCD = m\angle CDE$		*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$m\angle ABC = m\angle CDE$		*Transitive property*

2. In the figure at the right, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$.
   Prove that $b + d = 180$.



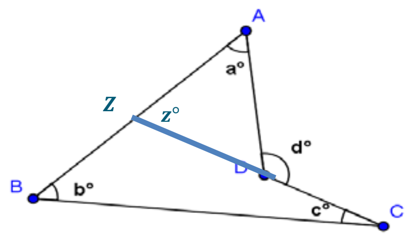
Label $c°$.

$b = c$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$c + d = 180$	*If parallel lines are cut by a transversal, then same-side interior angles are supplementary.*
$b + d = 180$	*Substitution property of equality*

3. In the figure at the right, prove that $d = a + b + c$.

Label Z and z.

$z = b + c$	*Exterior angle of a triangle equals the sum of the two interior opposite angles*
$d = z + a$	*Exterior angle of a triangle equals the sum of the two interior opposite angles*
$d = a + b + c$	*Substitution property of equality*



Exit Ticket (5 minutes)

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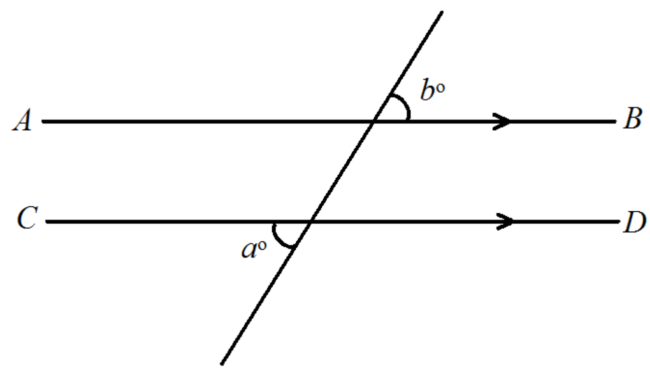
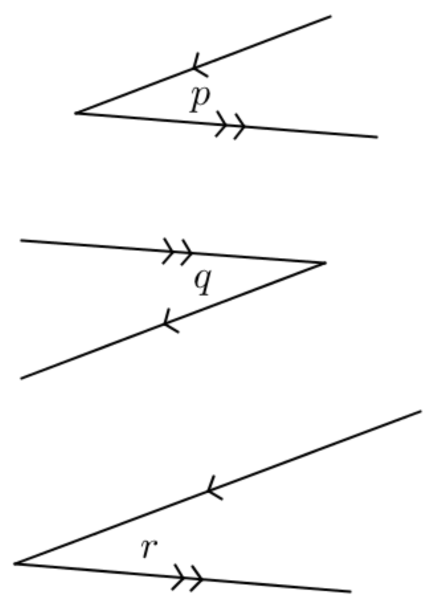
Exit Ticket

Write a proof for each question.

1. In the figure at the right, $\overline{AB} \parallel \overline{CD}$. Prove that $a = b$.



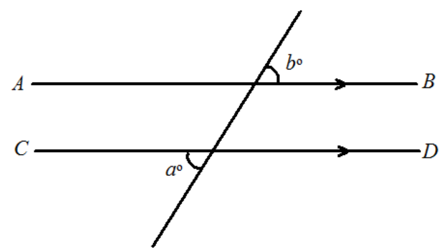
2. Prove $\text{m}\angle p = \text{m}\angle r$.



Exit Ticket Sample Solutions

Write a proof for each question.

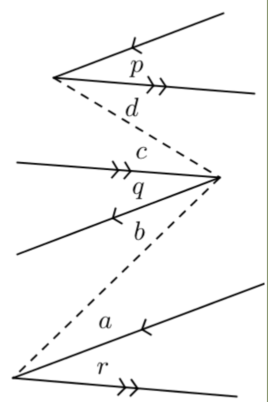
1. In the figure at the right, $\overline{AB} \parallel \overline{CD}$. Prove that $a = b$.



Write in angles c and d.

$a = c$	*Vertical angles are equal in measure.*
$c = d$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$d = b$	*Vertical angles are equal in measure.*
$a = b$	*Substitution property of equality*

2. Prove $\text{m}\angle p = \text{m}\angle r$.

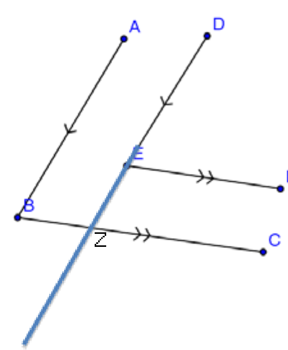


Mark angles a, b, c and d.

$\text{m}\angle p + \text{m}\angle d = \text{m}\angle c + \text{m}\angle q$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$\text{m}\angle d = \text{m}\angle c$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$\text{m}\angle p = \text{m}\angle q$	*Subtraction property of equality*
$\text{m}\angle q + \text{m}\angle b = \text{m}\angle a + \text{m}\angle r$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$\text{m}\angle a = \text{m}\angle b$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$\text{m}\angle q = \text{m}\angle r$	*Subtraction property of equality*
$\text{m}\angle p = \text{m}\angle r$	*Substitution property of equality*

Problem Set Sample Solutions

1. In the figure to the right, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$.
Prove that $\text{m}\angle ABC = \text{m}\angle DEF$.

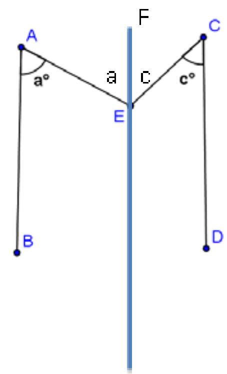


Extend DE through BC, and mark the intersection with BC as Z.

$\text{m}\angle ABC = \text{m}\angle EZC$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure*
$\text{m}\angle EZC = \text{m}\angle DEF$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure*
$\text{m}\angle ABC = \text{m}\angle DEF$	*Transitive property*

2. In the figure to the right, $\overline{AB} \parallel \overline{CD}$.

Prove that $m\angle AEC = a° + c°$.



Draw in line through E parallel to $\overline{AB}$ and $\overline{CD}$;

Add point F.

$m\angle BAE = m\angle AEF$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$m\angle DCE = m\angle FEC$	*If parallel lines are cut by a transversal, then alternate interior angles are congruent equal in measure*
$m\angle AEC = a° + c°$	*Angle addition postulate*